

## Conclusions of the MITP Workshop on T Violation and CPT Tests in Neutral-Meson Systems

K. R. Schubert<sup>1,2</sup>, L. Li Gioi<sup>3</sup>, A. J. Bevan<sup>4</sup> and A. Di Domenico<sup>5</sup>

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Participants: J. Bernabeu<sup>6</sup>, A. J. Bevan<sup>4</sup>, G. D'Ambrosio<sup>7</sup>, A. Denig<sup>2</sup>, A. Di Domenico<sup>5</sup>, H.-J. Gerber<sup>8</sup>, W. Gradl<sup>2</sup>, M. Heck<sup>9</sup>, T. Hurth<sup>2</sup>, J. S. Lange<sup>10</sup>, L. Li Gioi<sup>3</sup>, M. Neubert<sup>2</sup>, T. Ruf<sup>11</sup>, K. R. Schubert<sup>1,2</sup> and P. Villanueva-Perez<sup>6</sup>

The two-day workshop took place in the Institut für Kernphysik, Universität Mainz in a very lively and friendly atmosphere, and the participants thank MITP very much for providing the frame for our presentations and discussions. Half of the time was used for discussions, and in fact the workshop continued by a number of e-mail exchanges until summer 2013. This summary does not cover all contributions, but they are available on the Indigo page of the workshop [1]. The four parts of the summary are:

1. T Violation in Decays of Neutral B Mesons, K. R. Schubert
2. T and CPT studies in  $B^0\bar{B}^0$  transitions with Belle, L. Li Gioi
3. Future Measurements of T violation in B and D decays, A. J. Bevan
4. Direct tests of T and CPT symmetries in the entangled neutral Kaon system at a  $\Phi$  factory, A. Di Domenico

The first part covers the central point of the workshop, the interpretation of CP and T violation in the interplay of  $B^0\bar{B}^0$  transitions and decays  $B \rightarrow J/\psi K$ . It also covers the continued and very helpful discussions with H.-J. Gerber, T. Ruf, F. Martinez-Vidal, P. Villanueva-Perez and A. Di Domenico. Parts 2 to 4 cover the prospects of future experiments with K, D and B mesons.

The most sensitive tests of CPT symmetry remain the Bell-Steinberger analyses of the  $K^0\bar{K}^0$  system using unitarity which connects the CP-symmetry properties of all observed  $K_S$  and  $K_L$  decay modes with the CPT- and T-sensitive overlap  $\langle K_L | K_S \rangle$ . These analyses started in 1970 and have reached the impressive sensitivity of  $|m(\bar{K}^0) - m(K^0)| < 4 \cdot 10^{-19}$  GeV at 95% C.L. in 2012, as presented by G. D'Ambrosio at the workshop. An open question remains by how much invisible decays of neutral K mesons can influence the result. How well is unitarity tested experimentally? How much does the product of lifetime and the sum of all measured partial decay rates deviate from unity for  $K_S$  and  $K_L$  decays? And how much would the errors on  $\text{Re}\epsilon$  and  $\text{Im}\delta$  increase if the invisible modes would have maximal CP violation? As long as this is not answered quantitatively by experiments, “direct” tests of CPT symmetry remain important.

<sup>1</sup> Institut für Kern- und Teilchenphysik, Technische Universität Dresden, Germany

<sup>2</sup> Institut für Kernphysik, Johannes-Gutenberg-Universität Mainz, Germany

<sup>3</sup> Max-Planck-Institut für Physik, München, Germany

<sup>4</sup> Queen Mary, University of London, United Kingdom

<sup>5</sup> Dipartimento di Fisica, Sapienza Università di Roma, and INFN, Roma, Italy

<sup>6</sup> IFIC, Universitat de Valencia-CSIC, Valencia, Spain

<sup>7</sup> INFN, Napoli, Italy

<sup>8</sup> IPP, ETHZ, Zürich, Switzerland

<sup>9</sup> Institut für Experimentelle Kernphysik, Karlsruher Institut für Technologie, Karlsruhe, Germany

<sup>10</sup> Justus-Liebig-Universität, Giessen, Germany

<sup>11</sup> CERN, Geneva, Switzerland

# 1 T Violation in Decays of Neutral B Mesons

Klaus R. Schubert

Abstract: The CP-violating observable  $\text{Im } \lambda$  with  $\lambda = q\bar{A}(\bar{B}^0 \rightarrow J/\psi \bar{K}^0)/pA(B^0 \rightarrow J/\psi K^0)$  can be written as  $|\bar{A}/A| \cdot \text{Im} \tilde{\lambda}$ , where  $\tilde{\lambda} = q\bar{A}|A|/(pA|\bar{A}|)$ . In this product,  $|\bar{A}/A|$  is CPT violating and  $\text{Im} \tilde{\lambda}$  is T violating. Therefore, observation of  $\text{Im} \lambda \neq 0$  in the  $\sin \Delta mt$  term of the time-dependent rate of  $B^0 \rightarrow J/\psi K_S$  decays is a proof of T-symmetry violation. CPT violation in these decays would lead to a non-vanishing  $\cos \Delta mt$  term in the rate. The first measurements of  $\text{Im} \lambda \neq 0$  in 2001 already prove T violation. The BABAR 2012 analysis demonstrates that  $\text{Im} \lambda \neq 0$  leads to a “motion-reversal” difference in the rates for the transitions  $B^0, \bar{B}^0 \rightarrow B_+, B_-$  and  $B_+, B_- \rightarrow B^0, \bar{B}^0$ .

The notion “Time Reversal Violation” has two different meanings, either breaking the symmetry of the transformation  $t \rightarrow -t$  in the fundamental laws of an interaction, or unequal motions (in classical mechanics) or evolutions (in quantum mechanics) for two systems when exchanging initial and final state and reversing velocities and spins [2, 3]. I will use the term “T-symmetry violation” for the first and “motion-reversal violation” for the second case. In particle physics we use motion-reversal experiments, like the comparison of the two time-dependent rates for the transitions  $K^0 \rightarrow \bar{K}^0$  and  $\bar{K}^0 \rightarrow K^0$ , as a method to test T-symmetry of the weak interaction [4]. Since the recent BABAR experiment [5, 6] on motion reversal in the transitions  $B^0 \rightarrow B_-$  and  $B_- \rightarrow B^0$  has been a central discussion point in this workshop, this note summarizes its relevance for demonstrating T-symmetry violation.

$B^0 \bar{B}^0$  transitions are well known to be sensitive to the symmetries CP, T and CPT. In the Weisskopf-Wigner approximation, the evolution of the two-dimensional  $B^0$  state  $\Psi = \psi_1 B^0 + \psi_2 \bar{B}^0$  is given by

$$i\dot{\psi}_i = \Lambda_{ij}\psi_j,$$

where  $\Lambda_{ij} = m_{ij} - i\Gamma_{ij}/2$  with two time-independent hermitean 2x2 matrices. The general solution is

$$\psi_i(t) = U_{ij}(t)\psi_j(0),$$

where the time-dependent matrix  $U_{ij}(t)$  follows unambiguously from  $\Lambda_{ij}$  [7, 8]. The transition rates  $|\langle\psi_f|U|\psi_i\rangle|^2$  are determined by six real observables  $\Delta m = m_H - m_L$ ,  $\Gamma = (\Gamma_H + \Gamma_L)/2$ ,  $\Delta\Gamma = \Gamma_H - \Gamma_L$ ,

$$\text{Re}\delta + i \text{Im}\delta = \frac{(m_{22} - m_{11})/2 - i(\Gamma_{22} - \Gamma_{11})/4}{\Delta m - i\Delta\Gamma/2} \text{ and } \left|\frac{q}{p}\right| = \left|\sqrt{\frac{\Lambda_{21}}{\Lambda_{12}}}\right|.$$

A non-zero phase between  $m_{12}$  and  $\Gamma_{12}$  leads to  $|q/p| \neq 0$ , violating CP and T;  $\Lambda_{11} \neq \Lambda_{22}$  leads to  $\delta \neq 0$ , violating CP and CPT. All six observables have been measured [9], and  $\Delta\Gamma$ ,  $\text{Re}\delta$ ,  $\text{Im}\delta$  and  $|q/p| - 1$  are compatible with zero. Within experimental errors, the Hamiltonian for  $B^0 \bar{B}^0$  transitions is invariant under all transformations T, CPT and CP.

Decays of neutral B mesons into  $J\psi K_S$  and  $J\psi K_L$  are determined by only three more real observables, if they are dominated by a single weak amplitude,

$$|A| = |\langle J\psi K^0 | D | B^0 \rangle|, \quad |\bar{A}| = |\langle J\psi \bar{K}^0 | D | \bar{B}^0 \rangle|,$$

and  $\text{Im}\lambda$ , where  $\lambda$  is defined in a phase-convention independent way by

$$\lambda = \frac{q\bar{A}}{pA}. \quad (1)$$

The single-amplitude condition (no “direct T violation”) is assumed in this summary and can be tested in the data, as presented later. The operator  $D$  is the Hamiltonian for the decay, and

$|A|^2/\Gamma$  is the fraction of  $B^0 \rightarrow J/\psi K^0$  decays. The relevance of  $|\bar{A}/A|$  and  $\text{Im}\lambda$  is easily seen by introducing the factorization

$$\lambda = \left| \frac{\bar{A}}{A} \right| \cdot \tilde{\lambda} \text{ with } \tilde{\lambda} = \frac{q\bar{A}|A|}{pA|\bar{A}|} . \quad (2)$$

The observables  $|\bar{A}/A|$  and  $\tilde{\lambda}$  describe the CPT and T symmetry properties of the decays. If the matrix element  $\langle J/\psi K^0 | D | B^0 \rangle$  is given by a single weak amplitude, then CPT invariance of  $D$  requires  $|\bar{A}/A| = 1$  [10, 11], T invariance requires  $\text{Im}\tilde{\lambda} = 0$  [12, 11], and CP invariance requires  $|\bar{A}/A| = 1$  and  $\text{Im}\tilde{\lambda} = 0$ . Decays of neutral B mesons into flavour-specific states, e. g. into  $\ell^\pm \nu X$ , are given by only two more observables,

$$|A_\ell| = |\langle \ell^+ \nu X | D | B^0 \rangle| \text{ and } |\bar{A}_\ell| = |\langle \ell^- \nu X | D | \bar{B}^0 \rangle| .$$

The equivalent to  $\text{Im}\tilde{\lambda}$  is not present if the “ $\Delta Q = \Delta b$ ” rule is strictly valid, i. e.

$$\langle \ell^- \nu X | D | B^0 \rangle = \langle \ell^+ \nu X | D | \bar{B}^0 \rangle = 0 .$$

The following text consists of two parts. In the first part I present the consequences of CPT and T symmetry for the transitions  $B^0, \bar{B}^0 \rightarrow J/\psi K_S, K_L$  and  $B_-, B_+ \rightarrow B^0, \bar{B}^0$  as derived by H.-J. Gerber [11] in collaboration with M. Fidecaro and T. Ruf. In the second part I discuss the consequences for the 2001 analyses of BABAR [13] and Belle [14] and for the 2012 analysis of BABAR [5]. I will use some additional simplifying assumptions which have no influence on the main conclusions,  $|q/p| = 1$ ,  $\delta = 0$ ,  $\Delta\Gamma = 0$ ,

$$K_S = (K^0 + \bar{K}^0)/\sqrt{2}, \quad K_L = (K^0 - \bar{K}^0)/\sqrt{2}, \quad (3)$$

and validity of the “ $\Delta Q = \Delta b$ ” and “ $\Delta S = \Delta b$ ” rules in flavour-specific and  $J/\psi K$  decays, respectively. This leads to

$$A_S = A(B^0 \rightarrow J/\psi K_S) = A/\sqrt{2}, \quad \bar{A}_S = A(\bar{B}^0 \rightarrow J/\psi K_S) = \bar{A}/\sqrt{2},$$

$$A_L = A(B^0 \rightarrow J/\psi K_L) = A/\sqrt{2}, \quad \bar{A}_L = A(\bar{B}^0 \rightarrow J/\psi K_L) = -\bar{A}/\sqrt{2} .$$

The evolution operator  $U(t)$  is given by

$$U_{ij}(t) = e^{-\Gamma t/2} \begin{pmatrix} \cos(\Delta m t/2) & i \sin(\Delta m t/2) \cdot p/q \\ i \sin(\Delta m t/2) \cdot q/p & \cos(\Delta m t/2) \end{pmatrix}, \quad (4)$$

and  $\Delta S = \Delta b$  means that the decay matrix is diagonal,

$$D = \begin{pmatrix} A(B^0 \rightarrow J/\psi K^0) & A(\bar{B}^0 \rightarrow J/\psi K^0) \\ A(B^0 \rightarrow J/\psi \bar{K}^0) & A(\bar{B}^0 \rightarrow J/\psi \bar{K}^0) \end{pmatrix} = \begin{pmatrix} A & 0 \\ 0 & \bar{A} \end{pmatrix}. \quad (5)$$

The decay rate  $R = R(B^0 \rightarrow J/\psi K_S | t)$  of a  $B^0$  at time  $t = 0$  into the final state  $J/\psi K_S$  at time  $t$  is

$$\begin{aligned} R &= \frac{1}{2} e^{-\Gamma t} \left| \begin{pmatrix} A & \bar{A} \end{pmatrix} \begin{pmatrix} \cos(\Delta m t/2) & i \sin(\Delta m t/2) \cdot p/q \\ i \sin(\Delta m t/2) \cdot q/p & \cos(\Delta m t/2) \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right|^2 \\ &= |A|^2 \cdot \frac{e^{-\Gamma t}}{2} [1 + \kappa - \kappa \cdot \cos(\Delta m t) - \text{Im}\lambda \cdot \sin(\Delta m t)], \end{aligned} \quad (6)$$

with the CPT-violating parameter

$$\kappa = \frac{|\lambda|^2 - 1}{|\lambda|^2 + 1} \quad (7)$$

and  $\text{Im}\lambda$  from eq. 1. I also assume  $\kappa \ll 1$ , i. e.  $|\lambda|^2 = 1 + 2\kappa$ . If CPT is conserved,  $\kappa = 0$  and the cosine term has to vanish. If T is conserved,  $\text{Im}\lambda = 0$  and the sinus term has to vanish. If CP is conserved, the rate has to be a pure exponential. With good statistics and well-understood systematics, tests of CP, T and CPT can be made with a single time-dependent measurement of

the rate for  $B^0 \rightarrow J/\psi K_S^0$  decays. Additional rate measurements of  $B^0 \rightarrow J/\psi K_L^0$ ,  $\bar{B}^0 \rightarrow J/\psi K_S^0$  and  $\bar{B}^0 \rightarrow J/\psi K_L^0$  improve statistics as well as systematic uncertainties; but the full physics information is already contained in the time dependence of  $B^0 \rightarrow J/\psi K_S^0$  decays alone.

The derivation of the rate in eq. 6 allows a simple argument for the fact that  $\text{Im}(q\bar{A}/pA) \neq 0$  proves T violation of the Hamiltonian [15].  $\text{Im}(q\bar{A}/pA) \neq 0$  violates CP symmetry which implies CPT and/or T violation. CPT symmetry requires only  $|\bar{A}/A| = 1$  and nothing for the phase of  $\bar{A}/A$ . Since  $\text{Im}(q\bar{A}/pA) \neq 0$  does not contradict CPT symmetry, it must violate T. The same conclusion that  $\text{Im}\lambda \neq 0$  is CP- and T-violating is found by P. Villanueva-Perez [16].

The rates for  $B^0 \rightarrow J\psi K_L^0$ ,  $\bar{B}^0 \rightarrow J\psi K_S^0$  and  $\bar{B}^0 \rightarrow J\psi K_L^0$  follow from replacing  $(A, \bar{A})$  and  $(1, 0)$  in eq. 6 by  $(A, -\bar{A})$  and/or  $(0, 1)$ . All four rates are given by the same expression

$$R = |A|^2 \cdot \frac{e^{-\Gamma t}}{2} [1 + \kappa + C \cdot \cos(\Delta m t) + S \cdot \sin(\Delta m t)] , \quad (8)$$

with different values for the coefficients  $C$  and  $S$  as given in Table 1.

Table 1: Coefficients  $C$  and  $S$  for the rates in eq. 8.

	$C$	$S$
$B^0 \rightarrow J\psi K_S^0$	$-\kappa$	$-\text{Im}\lambda$
$B^0 \rightarrow J\psi K_L^0$	$-\kappa$	$+\text{Im}\lambda$
$\bar{B}^0 \rightarrow J\psi K_S^0$	$+\kappa$	$+\text{Im}\lambda$
$\bar{B}^0 \rightarrow J\psi K_L^0$	$+\kappa$	$-\text{Im}\lambda$

The T violation in all four rates is given by the same parameter  $\text{Im}\lambda$ , the CPT violation in all four by  $\kappa$ . From entangled  $B^0\bar{B}^0$  pairs with tagging by flavour-specific decays, the  $B^0$  rates obtain an extra factor  $|\bar{A}_\ell|^2$ , the  $\bar{B}^0$  rates  $|A_\ell|^2$ . The four rates are only equal, up to the signs in the table, if  $|\bar{A}_\ell| = |A_\ell|$ , i. e. CPT symmetry in flavour-specific decays.

With the Kaon sign-conventions in eq. 3, the states  $B_+$  and  $B_-$  [17, 18, 19],

$$B_+ = N(\bar{A}B^0 - A\bar{B}^0) , \quad B_- = N(\bar{A}B^0 + A\bar{B}^0) , \quad N^{-2} = |A|^2 + |\bar{A}|^2 = 2|A|^2(1 + \kappa) \quad (9)$$

have the properties that  $B_+$  decays only into  $J/\psi K_L^0$ , not into  $J/\psi K_S^0$ , and  $B_-$  only into  $J/\psi K_S^0$ , not into  $J/\psi K_L^0$ . Since the amplitudes in eq. 5 are defined as  $A = \langle J/\psi K^0 | D | B^0 \rangle$  and  $\bar{A} = \langle J/\psi \bar{K}^0 | D | \bar{B}^0 \rangle$ , the states in eq. 9 are ingoing states  $|B_+\rangle$  and  $|B_-\rangle$  and not the outgoing states in the transitions  $B^0, \bar{B}^0 \rightarrow B_+, B_-$ . With direct CPT violation, the two states are not orthogonal,  $\langle B_+ | B_- \rangle = \kappa$ . The orthogonal states to  $B_+$  and  $B_-$  are

$$B_{+\perp} = N(A^*B^0 + \bar{A}^*\bar{B}^0) \text{ and } B_{-\perp} = N(A^*B^0 - \bar{A}^*\bar{B}^0) , \quad (10)$$

respectively, and the normalization factor  $N$  is the same as in eq. 9. Independent of CPT invariance, an  $\Upsilon(4S)$  meson decays into the entangled two-body state

$$B^0\bar{B}^0 - \bar{B}^0B^0 = B_+B_{+\perp} - B_{+\perp}B_+ = B_-B_{-\perp} - B_{-\perp}B_- , \quad (11)$$

where the first  $B$  in this notation moves in direction  $\vec{p}$  and the second one in direction  $-\vec{p}$ . With CPT invariance,  $B_{+\perp} = B_-$  and  $B_{-\perp} = B_+$ . With  $\kappa \neq 0$ ,  $B_{+\perp}$  decays into both  $J\psi K_S$  and  $J\psi K_L$ . But if the first  $B$  decay of the entangled pair is  $J\psi K_S$ , then the remaining  $B$  is in the state  $B_+$ . The state  $B_{+\perp}$  is not used in the following motion-reversal discussion, its only purpose is the preparation of the state  $B_+$ . The analogous argument holds for  $B_{-\perp}$ . The decays  $\Upsilon(4S) \rightarrow B^0\bar{B}^0 \rightarrow (J/\psi K_S)(J/\psi K_S)$  and  $(J/\psi K_L)(J/\psi K_L)$  are forbidden in accordance with

Bose statistics. As a side remark, not relevant for T violation, the two states  $B_+$  and  $B_-$  are well defined physical states like the mixing eigenstates  $B_H$  and  $B_L$ , free of phase conventions, but all four are not CP eigenstates. The decay rates of  $B_+$  and  $B_-$  are

$$\Gamma_{\psi K} = \Gamma(B_- \rightarrow J\psi K_S) = \Gamma(B_+ \rightarrow J\psi K_L) = |A|^2(1 + \kappa) , \quad (12)$$

where the factor  $1 + \kappa$  originates from the normalization  $N$  in eq. 9. Dividing the rates  $R(t)$  in eq. 6 by this decay rate leads to

$$R_1(t) = \frac{e^{-\Gamma t}}{2} [1 + C_1 \cdot \cos(\Delta mt) + S_1 \cdot \sin(\Delta mt)] , \quad (13)$$

with the coefficients  $C_1$  and  $S_1$  as given in Table 2, using  $\text{Im}\tilde{\lambda} = \text{Im}\lambda/(1 + \kappa) = \text{Im}\lambda(1 - \kappa)$ . Note that these rates  $R_1(t) = R(t)/\Gamma_{\psi K}$  are not the rates for the transitions  $B^0, \bar{B}^0 \rightarrow B_+, B_-$  because of the difference between in- and outgoing states.

Calculation of the time-dependent rates  $R_2(t)$  for the transitions  $B^0, \bar{B}^0 \rightarrow B_+, B_-$  requires the outgoing states

$$\langle B_+ | = N(\bar{A}^* \langle B^0 | - A^* \langle \bar{B}^0 |) , \quad \langle B_- | = N(\bar{A}^* \langle B^0 | + A^* \langle \bar{B}^0 |) , \quad (14)$$

leading to

$$R_2(t) = \frac{e^{-\Gamma t}}{2} [1 + C_2 \cdot \cos(\Delta mt) + S_2 \cdot \sin(\Delta mt)] , \quad (15)$$

with  $C_2$  and  $S_2$  in Table 2. The motion-reversed transitions  $B_+, B_- \rightarrow B^0, \bar{B}^0$  require the ingoing states of  $B_+$  and  $B_-$ . Their rates  $R_3(t)$  are found to be

$$R_3(t) = \frac{e^{-\Gamma t}}{2} [1 + C_3 \cdot \cos(\Delta mt) + S_3 \cdot \sin(\Delta mt)] , \quad (16)$$

with  $C_3$  and  $S_3$  in Table 2.

Table 2: Coefficients  $C_i$  and  $S_i$ ,  $i = 1$  for the rates  $R(B \rightarrow J/\psi K)/\Gamma_{\psi K}$  in eq. 13,  $i = 2$  for the transitions in eq. 15, and  $i = 3$  for the transitions in eq. 16.

	$C_1$	$S_1$		$C_2$	$S_2$		$C_3$	$S_3$
$B^0 \rightarrow J/\psi K_S$	$-\kappa$	$-\text{Im}\tilde{\lambda}$	$B^0 \rightarrow B_-$	$+\kappa$	$-\text{Im}\tilde{\lambda}$	$B_- \rightarrow B^0$	$+\kappa$	$+\text{Im}\tilde{\lambda}$
$B^0 \rightarrow J/\psi K_L$	$-\kappa$	$+\text{Im}\tilde{\lambda}$	$B^0 \rightarrow B_+$	$+\kappa$	$+\text{Im}\tilde{\lambda}$	$B_+ \rightarrow B^0$	$+\kappa$	$-\text{Im}\tilde{\lambda}$
$\bar{B}^0 \rightarrow J/\psi K_S$	$+\kappa$	$+\text{Im}\tilde{\lambda}$	$\bar{B}^0 \rightarrow B_-$	$-\kappa$	$+\text{Im}\tilde{\lambda}$	$B_- \rightarrow \bar{B}^0$	$-\kappa$	$-\text{Im}\tilde{\lambda}$
$\bar{B}^0 \rightarrow J/\psi K_L$	$+\kappa$	$-\text{Im}\tilde{\lambda}$	$\bar{B}^0 \rightarrow B_+$	$-\kappa$	$-\text{Im}\tilde{\lambda}$	$B_+ \rightarrow \bar{B}^0$	$-\kappa$	$+\text{Im}\tilde{\lambda}$

The time-dependent motion-reversal asymmetries like between  $B^0 \rightarrow B_-$  and  $B_- \rightarrow B^0$ ,

$$\mathcal{A}_{MR} = \frac{R_3(t) - R_2(t)}{R_3(t) + R_2(t)} = \frac{C_3 - C_2}{2} \cdot \cos(\Delta mt) + \frac{S_3 - S_2}{2} \cdot \sin(\Delta mt) , \quad (17)$$

are sensitive to only T-symmetry violation;  $(C_3 - C_2)/2 = 0$  and  $(S_3 - S_2)/2 = S_3 = \pm \text{Im}\tilde{\lambda}$ . The time-dependent quasi-motion-reversal asymmetries like between  $B^0 \rightarrow J/\psi K_S$  and  $B_- \rightarrow B^0$ ,

$$\mathcal{A}_{QMR} = \frac{R_3(t) - R_1(t)}{R_3(t) + R_1(t)} = \frac{C_3 - C_1}{2} \cdot \cos(\Delta mt) + \frac{S_3 - S_1}{2} \cdot \sin(\Delta mt) , \quad (18)$$

are sensitive to both CPT- and T-symmetry violation;  $(C_3 - C_1)/2 = C_3 = \pm \kappa$  and  $(S_3 - S_1)/2 = S_3 = \pm \text{Im}\tilde{\lambda}$ .

The CP-violation analyses [13, 14] have combined the transitions  $B^0, \bar{B}^0 \rightarrow J\psi K_S, K_L$  and  $B_-, B_+ \rightarrow B^0, \bar{B}^0$  by using events with both signs of  $\Delta t = t(\text{decay to } J/\psi K_{S,L}) - t(\text{decay to } \ell^\pm X)$ . Recalling that  $\Delta t = +t$  and  $\Delta t = -t$  for the events with  $i = 1$  and  $i = 3$  in Table 2 respectively, and inspecting the signs of  $C_1$  and  $C_3$ , shows that the determination of  $|\lambda|$  in these analyses is also a determination of  $\kappa$ . Accidental cancellations between direct T violation (two decay amplitudes with different phases giving leading to  $|\lambda| \neq 1$ ) and direct CPT violation ( $\kappa \neq 0$ ) are very unlikely. Therefore, the analyses determine  $\text{Im}\bar{\lambda}$  and  $\kappa$  and show T-symmetry violation with a significance of about  $4\sigma$  independent of any assumption on CPT [11].

The recent BABAR analysis [5] has for the first time separated the CP-violation data into events with  $\Delta t > 0$  and  $\Delta t < 0$  and has determined eight uncorrelated coefficients  $C_i$  and  $S_i$  for the  $\cos \Delta mt$  and  $\sin \Delta mt$  terms. All  $C_i$  values are compatible with CPT symmetry, and all  $S_i$  values prove T-symmetry violation. Since earlier analyses can lead to the same conclusion, the main merit of the analysis in ref. [5] is its demonstration of T-symmetry violation by motion-reversal violation. The data can only use quasi-motion-reversals, comparing e. g.  $B^0 \rightarrow J/\psi K_S$  and  $B_- \rightarrow B^0$ , and not motion-reversals with  $B^0 \rightarrow B_-$  and  $B_- \rightarrow B^0$ . However, the null results for the eight determinations of  $\kappa$  justify this approximation.

## References

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## 2 T and CPT studies in $B^0\bar{B}^0$ transition with Belle

Luigi Li Gioi

The Belle collaboration, having performed experiments at the KEKB B-factory since 1999, made the essential observations of the CP violation, proving the differences in the decays of B mesons compared to their anti-particles  $\bar{B}^0$ .

The KEKB B-factory is currently under upgrade to a new generation of super flavor factory (SuperKEKB) which aims to deliver more than  $50\text{ ab}^{-1}$  by the end of 2022. Together with the machine, the Belle detector upgrade is ongoing (Belle II) [2]. The main redesign goals are to cope with the much higher physics rates and the much larger background, as well as improving the overall physics performance. For T and CPT violation studies the new tracking system plays a key role. The tracking system in the former Belle detector consisted of 4 layer of Si strip vertex detector (SVD), followed by a Central Drift Chamber (CDC). For the new tracking system a two-layer pixel detector (PXD) for the innermost Si layers will be implemented. The SVD will be replaced entirely as well as the CDC: due to the harsh backgrounds the inner radius of the CDC has to be moved out and the two outer layers of the new SVD will cover the gap. The momentum resolution of charged particles will be improved by extending the CDC to a larger radius.

The impact parameters:  $d_0$  and  $z_0$ , defined as the projections of the distance from the point of closest approach to the origin, are a good measure of the overall performance of the tracking system and as such are used to find the optimal tracker configuration. An improvement of roughly a factor two is expected on the impact parameter resolution. The introduction of a new vertex fitter together with the improvement of the alignment procedure will thus sensibly improve the systematic error of any time dependent measurement.

### 2.1 CPT Violation

In the presence of CPT violation, the flavor and mass eigenstates of the neutral B mesons are related by  $|B_L\rangle = p\sqrt{(1-z)}|B^0\rangle + q\sqrt{(1+z)}|\bar{B}^0\rangle$  and  $|B_H\rangle = p\sqrt{(1+z)}|B^0\rangle - q\sqrt{(1-z)}|\bar{B}^0\rangle$  where  $|B_L\rangle$  and  $|B_H\rangle$  are the light and heavy mass eigenstates. Here  $z$  is a complex parameter accounting for CPT violation; CPT is violated if  $z \neq 0$ . Then, the time dependent decay rate of the two B mesons generated from the  $\Upsilon(4S) \rightarrow B^0\bar{B}^0$  is given by [3]:

$$P(\Delta t, f_{rec}, f_{tag}) = \frac{\Gamma_d}{2} e^{-\Gamma_d |\Delta t|} \left[ \frac{|\eta_+|^2 + |\eta_-|^2}{2} \cosh\left(\frac{\Delta\Gamma_d}{2} \Delta t\right) - \text{Re}(\eta_+^* \eta_-) \sinh\left(\frac{\Delta\Gamma_d}{2} \Delta t\right) \right. \\ \left. + \frac{|\eta_+|^2 - |\eta_-|^2}{2} \cos(\Delta m_d \Delta t) + \text{Im}(\eta_+^* \eta_-) \sin(\Delta m_d \Delta t) \right] \quad (1)$$

where  $\eta_+ = A_1 \bar{A}_2 - \bar{A}_1 A_2$ ,  $\eta_- = \sqrt{(1-z^2)}(p/q A_1 A_2 - q/p \bar{A}_1 \bar{A}_2) + z(A_1 \bar{A}_2 + \bar{A}_1 A_2)$ ,  $A_1$  and  $A_2$  are the decay amplitudes of the reconstruction and tag side B mesons to  $f_1$  and  $f_2$  final states.

The Belle collaboration measured the CPT-violating parameter  $z$  and the normalized total decay-width difference  $\Delta\Gamma_d/\Gamma_d$  [4] in  $B^0 \rightarrow J/\psi K^0$  ( $K^0 = K_S, K_L$ ),  $B^0 \rightarrow D^{(*)-} h^+$  ( $h^+ = \pi^+$  for  $D^-$  and  $\pi^+, \rho^+$  for  $D^{*-}$ ), and  $B^0 \rightarrow D^{*-} l^+ \nu_l$  ( $l^+ = e^+, \mu^+$ ) decays. Most of the sensitivity to  $\text{Re}(z)$  and  $\Delta\Gamma_d/\Gamma_d$  is obtained from neutral B-meson decays to  $f_{CP}$ , while  $\text{Im}(z)$  is constrained primarily from other neutral B-meson decay modes. The results are based on a data sample of  $535 \times 10^6 B\bar{B}$  pairs collected at the  $\Upsilon(4S)$  resonance:  $\text{Re}(z) = [+1.9 \pm 3.7(\text{stat}) \pm 3.3(\text{syst})] \times 10^{-2}$ ,  $\text{Im}(z) = [-5.7 \pm 3.3(\text{stat}) \pm 3.3(\text{syst})] \times 10^{-3}$  and  $\Delta\Gamma_d/\Gamma_d = [-1.7 \pm 1.8(\text{stat}) \pm 1.1(\text{syst})] \times 10^{-2}$ , all of which are consistent with zero. This is the most precise single measurement of these parameters in the neutral B-meson system to date. The dominant contributions to the systematic uncertainties are from vertex reconstruction and the tag-side interference (TSI) [5]; the next largest contributions are from fit bias. Using the new tracking system of the Belle II detector and refining the analysis will then permit to sensibly reduce the amount of the systematic uncertainty and to fully exploit the

larger data sample that will be collected by the Belle II experiment. Assuming a final integrated luminosity of  $50 \text{ ab}^{-1}$ , an improvement of  $\sqrt{50}/0.5 = 10$  is expected.

## 2.2 T Violation

The CPLEAR collaboration reported on the observation of time-reversal symmetry violation through a comparison of the probabilities of  $K^0$  transforming into  $\bar{K}^0$  and  $\bar{K}^0$  into  $K^0$  as a function of the neutral-kaon eigentime  $t$ . An average decay-rate asymmetry  $\langle A_T \rangle = [6.6 \pm 1.3(\text{stat}) \pm 1.0(\text{syst})] \times 10^{-3}$  was measured over the interval  $1\tau_S < \tau < 20\tau_S$  [6].

In the case of the  $B^0\bar{B}^0$  mesons the asymmetry is then expected to be close to zero; a value significantly larger than  $10^{-3}$  would be an indication of new physics [7].

The Belle collaboration performed this measurement using the semileptonic decays of the neutral  $B$  meson [8]: a possible difference between the transitions  $B^0 \rightarrow \bar{B}^0$  and  $\bar{B}^0 \rightarrow B^0$  can manifest itself as a charge asymmetry in same-sign dilepton events in  $\Upsilon(4S)$  decays when prompt leptons from semileptonic decays of neutral B mesons are selected. Using a data sample of  $78 \text{ fb}^{-1}$  recorded at the  $\Upsilon(4S)$  resonance and  $9 \text{ fb}^{-1}$  recorded at an energy 60 MeV below the resonance, Belle measures  $A_{sl} = [1.1 \pm 7.9(\text{stat}) \pm 8.5(\text{syst})] \times 10^{-3}$ . The dominant contributions to the systematic uncertainties are from Track finding efficiency and Continuum subtraction; they can be reduced with a better knowledge of the tracking system and a refinement of the analysis.

Recently the BaBar collaboration reported a measurement of T-violating parameters in the time evolution of neutral B mesons [9] using the decays of entangled neutral B mesons into definite flavor states ( $B^0$  or  $\bar{B}^0$ ), and  $J/\psi K_L$  or  $c\bar{c}K_S$  final states with the comparisons between the probabilities of four pairs of T-conjugated transitions as a function of the time difference between the two B decays [10]. The results obtained by the BaBar collaboration are in agreement with CPT conservation.

At this point the Belle collaboration would not expect that repeating the BaBar analysis would improve the results significantly. However there is an interest in these results as a test of CPT.

The experimental method consists in dividing the very same  $\Delta t$  distribution used in the standard CP analysis in two parts:  $\Delta t > 0$  and  $\Delta t < 0$ , and fitting them using the same function used for the CP violation analysis:

$$g_{\alpha,\beta}^{\pm}(\Delta\tau) = e^{-\Gamma_d\Delta\tau} \{1 + S_{\alpha,\beta}^{\pm} \sin(\Delta m_d \Delta\tau) + C_{\alpha,\beta}^{\pm} \cos(\Delta m_d \Delta\tau)\} \quad (2)$$

where  $\Delta\Gamma_d = 0$  is assumed, the indices  $\alpha = l^+l^-$  (flavor state),  $\beta = K_S, K_L$  (CP states) and the symbol  $+$  or  $-$  indicates whether the decay to the flavor final state  $\alpha$  occurs before or after the decay to the CP final state  $\beta$ .  $\Gamma_d$  is the average decay width and  $\Delta m_d$  is the mass difference between the neutral B mass eigenstates. Since, using the data sample collected by the Belle experiment in the standard CP violation analysis [11], no tension is observed in the fit of the  $\Delta t$  distributions between the left and right sides of the distributions and between  $J/\psi K_L$  and  $c\bar{c}K_S$ , this analysis could not yield results compatible with CPT violation.

The measurement of the T-violation parameters could become important for the Belle II collaboration, when a very high experimental precision will be achieved. At that time also a number of measurements should be accessible to test T symmetry invariance in  $b \rightarrow u, d$  and  $s$  transitions as well as in the charm sector to test  $c \rightarrow d$  and  $c \rightarrow s$  transitions [12]. It is also considered an advantage that any CP and T measurement should yield about the same result if CPT is conserved. This would allow a cross check of any CP measurement.



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### 3 Future Measurements of $T$ violation in $B$ and $D$ decays

Adrian J. Bevan

Abstract: The potential for future measurements of  $T$  violation in  $B$  and  $D$  decays is summarised here. This discussion considers possible quark flavour changing transitions from  $b$  and  $c$  quarks to all kinematically accessible final states via tree and loop topologies.

There is scope to extend the tests of the  $T$  symmetry using quantum entangled pairs of neutral mesons outlined in Refs [1, 2, 3], and performed recently by *BABAR* [4], to other  $CP$  filter final states of  $B$  decays as well as for some charm decays. These measurements can be used to test the nature of the  $T$  symmetry under quark transitions at tree and loop level in  $b \rightarrow c$ ,  $s$ ,  $u$ , and  $d$  transitions, as well as  $c \rightarrow d$  and  $s$  transitions. The loop transition  $c \rightarrow u$  is experimentally inaccessible given that the corresponding loop is Cabibbo suppressed (i.e. by a factor of  $|V_{ub}| \propto \lambda^3$ ) and more copious loop transitions will dominate any attempt to extract that term. There are two categories of orthonormal  $CP$  basis filters that can be used for such measurements: (i) the approximately orthonormal pair of  $T$ -conjugate decays  $X + K_S^0$  and  $X + K_L^0$ , and (ii)  $T$  self-conjugate decays of a pseudoscalar to two spin-one particles where a transversity analysis of the final state allows one to experimentally distinguish between  $CP$  even and  $CP$  odd parts of the decay. This summary is based on Ref. [5] and naive numerical estimates prepared for this workshop. These estimates are obtained using existing experimental results on time-dependent  $CP$  asymmetry measurements from the  $B$  Factories. Only uncertainties are quoted as it would be incorrect to extrapolate central values for the  $CP$  asymmetry parameters in terms of the  $T$  asymmetry parameters  $\Delta S_T^\pm$  without re-analysing the data.

#### 3.1 $B$ decay measurements at the $\Upsilon(4S)$

The *BABAR* and Belle experiments already have data that can be used to perform a number of  $T$  violation tests beyond the  $b \rightarrow c$  transitions described in Ref. [4]. In the near future Belle II is expected to accumulate  $50 \text{ ab}^{-1}$  of data which will enable one to perform precision measurements of time-dependent  $T$ -violating asymmetries that can be related to the CKM matrix description of quark flavor changing transitions.  $T$  violation has been established in entangled  $B$  meson decays to combinations of flavor tagged final states denoted by  $\ell^\pm X$  and  $CP$  filter final states  $B_+$  and  $B_-$ , taken to be  $c\bar{c}K_{S,L}$ . The presence of two sets of orthonormal filters enables one to test  $T$ ,  $CP$  and  $CPT$  via the different pairings of events as a function of proper time difference  $\Delta\tau$  as outlined by Bernabeu et al. In analogy with the  $b \rightarrow c$  transitions proposed by Bernabeu et al., it is possible to study  $T$ -violation in the interference between mixing and decay amplitudes in  $b \rightarrow s$  transitions involving  $\eta' K_{S,L}$ ,  $\phi K_{S,L}$  and  $\omega K_{S,L}$   $CP$  filter pairs, however the  $B$  Factories only have sufficient data for the first two measurements as  $B \rightarrow \omega K_L^0$  has not yet been observed. Nonetheless one can estimate the anticipated precision attainable on a test of  $T$  using the  $\omega K_{S,L}$   $CP$  filters.

The set of  $B_+$  and  $B_-$  involving  $K_S^0$  and  $K_L^0$  is approximately orthonormal (this is a good approximation given current levels of experimental precision). As mentioned above it is also possible to define an exactly orthonormal basis pair for the  $B_+$  and  $B_-$  filters as illustrated in the following. The  $b \rightarrow c$  transition  $B \rightarrow J/\psi K^{*0}$  is composed of three  $P$  wave parts, one is a longitudinal component that is  $CP$  even and there are two transverse components: the perpendicular ( $CP$  odd) and parallel ( $CP$  even) parts. As a result it is possible to define an exactly orthonormal basis of  $B_+$  and  $B_-$  decays into the  $J/\psi K^{*0}$  final state (the same is true for other decays of a neutral pseudo-scalar meson to two spin-one particles). In analogy with this discussion, one can also test the  $T$  symmetry using the  $CP$  even and odd parts of other modes, such as the decay  $B \rightarrow D^* D^*$  (a  $b \rightarrow d$  quark transition),  $B \rightarrow \phi K^*$  (a  $b \rightarrow s$  quark transition), and  $B \rightarrow \rho^0 \rho^0$  (a  $b \rightarrow u$  quark transition) as a basis of  $CP$  filters.

Estimates of the experimental precisions attainable for these modes, where possible to compute, can be found in Table 1. It is likely that Belle II will be able to observe  $T$  violation at a significance

of at least  $5\sigma$  for all of the modes listed, assuming that the violation occurs at the level expected within the SM. Given that  $\Delta S_T^\pm$  is related to  $\lambda_f = (q/p)(\bar{A}/A)$ , one can relate these modes to the angles of the Unitarity triangle, and in this case (for  $b \rightarrow c$ ,  $d$ , and  $s$  transitions) one has a set of tree and loop dominated measurements of  $\Delta S_T^\pm = \mp 2 \sin 2\beta \simeq \mp 1.36$ , where the cosine coefficient  $\Delta C_T^\pm$  is expected to be zero in the SM. The decay  $B \rightarrow \rho^0 \rho^0$  can be used to measure the Unitarity triangle  $\alpha$  in the SM. There is insufficient experimental data to extrapolate the precision with which one may be able to constrain  $\Delta S_T^\pm$  in  $B \rightarrow \rho^0 \rho^0$ , however such a measurement is feasible at Belle II.

Table 1: Estimates of the precisions on the  $T$  asymmetry parameter  $\Delta S_T^\pm$  for different experiments, based on existing results from the  $B$  Factories.

$CP$ filters	$\sigma(\Delta S_T^\pm)$ <i>BABAR</i> / Belle	$\sigma(\Delta S_T^\pm)$ Belle II
$\eta' K^0$	0.56	0.08
$\phi K^*$	1.14	0.13
$\phi K^0$	1.84	0.17
$\omega K^0$	1.95	0.22
$D^* D^*$	2.0	0.29

### 3.2 $D$ decay measurements at an asymmetric energy $\tau$ -charm experiment

A number of time-dependent  $CP$  asymmetry measurements in charm have been proposed in order to measure the phase of mixing, and to constrain the  $cu$  Unitarity triangle angle  $\beta_c$  (which should be small in the Standard Model) [6]. Neutral  $D$  meson pairs created at the charm threshold,  $\psi(3770)$ , decay in an entangled state in analogy with neutral  $B$  mesons at the  $\Upsilon(4S)$ . As a result there are a number of  $D \rightarrow XK_{S,L}$  states and a number of final states composed of two spin-one particles that can be used to test the  $T$  symmetry in the charm sector. These can be classified into those states that measure the phase of charm mixing (via a  $c \rightarrow s$  transition), and those that measure a combination of the mixing phase and  $\beta_c$  (via a  $c \rightarrow d$  transition). Decays of the first type include  $\phi K_{S,L}$  and  $\pi^+ \pi^- K_{S,L}$  final states, whereas decays of the second type include  $K^+ K^- K_{S,L}$  and  $\rho^0 \rho^0$  final states.

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## 4 Direct test of T and CPT symmetries in the entangled neutral kaon system at a $\phi$ -factory

Antonio Di Domenico

### 4.1 Introduction

In this note the possibility to perform direct tests of T and CPT symmetries in the neutral kaon system is discussed. Here *direct test* means a test whose outcome is independent from the result of any other discrete symmetry test, as discussed in Refs.[1, 2, 3, 4, 5]. In order to implement such direct tests it has been suggested to exploit the Einstein-Podolsky-Rosen (EPR) entanglement of neutral mesons produced at a  $\phi$ -factory (or B-factory) [2, 3, 4, 5]. In fact in this case the initial state of the neutral kaon pair produced in  $\phi \rightarrow K^0 \bar{K}^0$  decay can be rewritten in terms of any pair of orthogonal states  $|K_+\rangle$  and  $|K_-\rangle$ :

$$|i\rangle = \frac{1}{\sqrt{2}}\{|K^0\rangle|\bar{K}^0\rangle - |\bar{K}^0\rangle|K^0\rangle\} = \frac{1}{\sqrt{2}}\{|K_+\rangle|K_-\rangle - |K_-\rangle|K_+\rangle\}. \quad (1)$$

Here one can consider the states  $|K_+\rangle$ ,  $|K_-\rangle$  defined as follows:  $|K_+\rangle$  is the state filtered by the decay into  $\pi\pi$  ( $\pi^+\pi^+$  or  $\pi^0\pi^0$ ), a pure CP = +1 state; analogously  $|K_-\rangle$  is the state filtered by the decay into  $3\pi^0$ , a pure CP = -1 state. Their orthogonal states correspond to the states which cannot decay into  $\pi\pi$  or  $3\pi^0$ , defined, respectively, as

$$\begin{aligned} |\tilde{K}_-\rangle &\propto [|K_L\rangle - \eta_{\pi\pi}|K_S\rangle] \\ |\tilde{K}_+\rangle &\propto [|K_S\rangle - \eta_{3\pi^0}|K_L\rangle], \end{aligned} \quad (2)$$

with  $\eta_{\pi\pi} = \langle\pi\pi|T|K_L\rangle/\langle\pi\pi|T|K_S\rangle$  and  $\eta_{3\pi^0} = \langle 3\pi^0|T|K_S\rangle/\langle 3\pi^0|T|K_L\rangle$ . With these definitions of states, it can be shown that the condition of orthogonality  $\langle K_-|K_+\rangle = 0$ , (i.e.  $|K_+\rangle \equiv |\tilde{K}_+\rangle$  and  $|K_-\rangle \equiv |\tilde{K}_-\rangle$ ) corresponds to assume negligible direct CP (or CPT) violation contributions (i.e.  $\epsilon', \epsilon'_{000} \ll \epsilon$ ), assumption quite well satisfied for neutral kaons (see detailed discussion in Appendix A of Ref. [5]). The validity of the  $\Delta S = \Delta Q$  rule is also assumed, so that the two flavor orthogonal eigenstates  $|K^0\rangle$  and  $|\bar{K}^0\rangle$  are identified by the charge of the lepton in semileptonic decays, i.e. a  $|K^0\rangle$  can decay into  $\pi^-\ell^+\nu$  and not into  $\pi^+\ell^-\bar{\nu}$ , and vice-versa for a  $|\bar{K}^0\rangle$ .

Thus, exploiting the perfect anticorrelation of the states implied by Eq. (1), it is possible to have a “flavor-tag” or a “CP-tag”, i.e. to infer the flavor ( $K^0$  or  $\bar{K}^0$ ) or the CP ( $K_+$  or  $K_-$ ) state of the still alive kaon by observing a specific flavor decay ( $\pi^+\ell^-\nu$  or  $\pi^-\ell^+\bar{\nu}$ ) or CP decay ( $\pi\pi$  or  $\pi^0\pi^0\pi^0$ ) of the other (and first decaying) kaon in the pair. In this way one can experimentally access the transitions listed in Table 1, which can be divided into four categories of events, corresponding to independent T, CP and CPT tests.

Reference	T-conjug.	CP-conjug.	CPT-conjug.
$K^0 \rightarrow K_+$	$K_+ \rightarrow K^0$	$\bar{K}^0 \rightarrow K_+$	$K_+ \rightarrow \bar{K}^0$
$K^0 \rightarrow K_-$	$K_- \rightarrow K^0$	$\bar{K}^0 \rightarrow K_-$	$K_- \rightarrow \bar{K}^0$
$\bar{K}^0 \rightarrow K_+$	$K_+ \rightarrow \bar{K}^0$	$K^0 \rightarrow K_+$	$K_+ \rightarrow K^0$
$\bar{K}^0 \rightarrow K_-$	$K_- \rightarrow \bar{K}^0$	$K^0 \rightarrow K_-$	$K_- \rightarrow K^0$

Table 1: Scheme of possible reference transitions and their associated T, CP or CPT conjugated processes accessible at a  $\phi$ -factory.

## 4.2 T symmetry test

For the direct T symmetry test one can define the following ratios of probabilities:

$$\begin{aligned} R_1(\Delta t) &= P[K^0(0) \rightarrow K_+(\Delta t)] / P[K_+(0) \rightarrow K^0(\Delta t)] \\ R_2(\Delta t) &= P[K^0(0) \rightarrow K_-(\Delta t)] / P[K_-(0) \rightarrow K^0(\Delta t)] \\ R_3(\Delta t) &= P[\bar{K}^0(0) \rightarrow K_+(\Delta t)] / P[K_+(0) \rightarrow \bar{K}^0(\Delta t)] \\ R_4(\Delta t) &= P[\bar{K}^0(0) \rightarrow K_-(\Delta t)] / P[K_-(0) \rightarrow \bar{K}^0(\Delta t)] . \end{aligned} \quad (3)$$

The measurement of any deviation from the prediction  $R_i(\Delta t) = 1$  imposed by T invariance is a signal of T violation. At a  $\phi$ -factory the corresponding observable quantities are two ratios,  $R_2^{\text{exp}}(\Delta t)$  and  $R_4^{\text{exp}}(\Delta t)$  (see Fig.1), of double decay rates as a function of the difference of kaon decay times  $\Delta t$  [6, 5]; for  $\Delta t > 0$  one has (the first decay product symbol in parenthesis indicates the first in time of the two kaon decays):

$$R_2^{\text{exp}}(\Delta t) \equiv \frac{I(\ell^-, 3\pi^0; \Delta t)}{I(\pi\pi, \ell^+; \Delta t)} = R_2(\Delta t \times D) \quad , \quad R_4^{\text{exp}}(\Delta t) \equiv \frac{I(\ell^+, 3\pi^0; \Delta t)}{I(\pi\pi, \ell^-; \Delta t)} = R_4(\Delta t \times D) \quad (4)$$

while for  $\Delta t < 0$ :

$$R_2^{\text{exp}}(\Delta t) = \frac{D}{R_3(|\Delta t|)} \quad , \quad R_4^{\text{exp}}(\Delta t) = \frac{D}{R_1(|\Delta t|)} . \quad (5)$$

Here the normalization constant  $D$ , assuming no CPT violation in semileptonic decays, is  $D = \{\text{BR}(K_L \rightarrow 3\pi^0) \cdot \Gamma_L\} / \{\text{BR}(K_S \rightarrow \pi\pi) \cdot \Gamma_S\}$ .

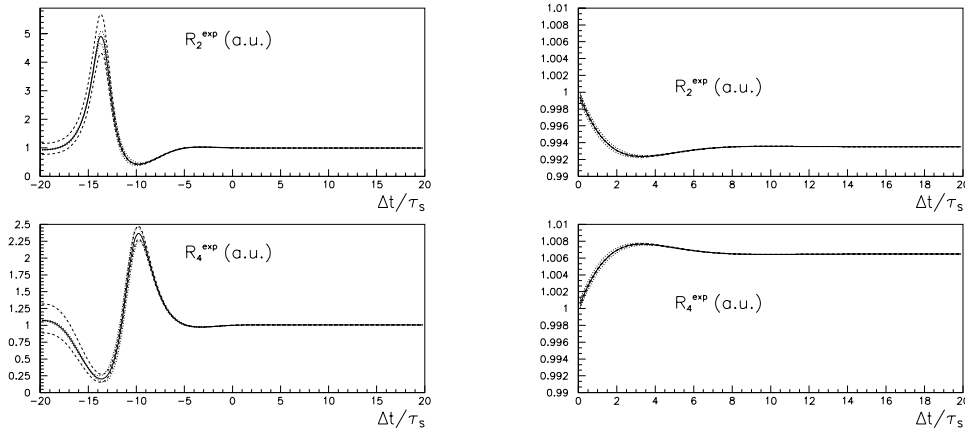


Figure 1: The expected ratios  $R_2^{\text{exp}}(\Delta t)$  (left top) and  $R_4^{\text{exp}}(\Delta t)$  (left bottom) as a function of  $\Delta t$  (solid line); dashed lines correspond to  $\pm 10\%$  uncertainty on  $\eta_{3\pi^0}$ ; the constant  $D$  has been fixed to one for simplicity. Zoomed plots for  $\Delta t > 0$  (right).

The KLOE-2 experiment at DAΦNE with an integrated luminosity of  $\mathcal{O}(10 \text{ fb}^{-1})$  [7] could make a statistically significant T test, measuring the ratios  $R_2^{\text{exp}}(\Delta t)$  and  $R_4^{\text{exp}}(\Delta t)$  integrated in the

statistically most populated  $\Delta t$  region,  $0 \leq \Delta t \leq 300 \tau_S$  [5]. Unfortunately in this region  $R_2^{\text{exp}}(\Delta t)$  and  $R_4^{\text{exp}}(\Delta t)$  are expected to be constant (see Fig.1 right), and a precise knowledge of the normalization  $D$  is needed in order to detect T violation. This also implies that T violation in this  $\Delta t$  region is constant, proportional to  $\Re\epsilon$ , i.e. directly proportional to  $\Delta\Gamma$ , the width difference of the mass eigenstates. Therefore, in this  $\Delta t$  region, T violation would not be present in the limit  $\Delta\Gamma \rightarrow 0$ , as one might like [1].

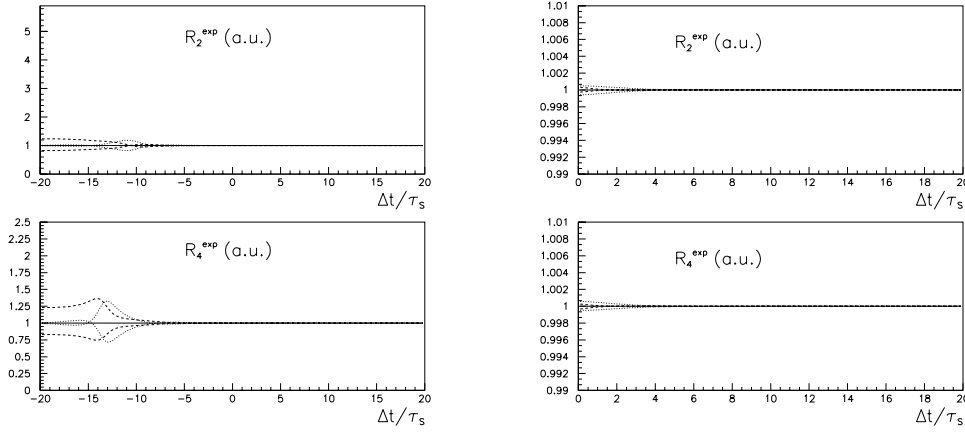


Figure 2: The expected ratios  $R_{2,CPT}^{\text{exp}}(\Delta t)$  (left top) and  $R_{4,CPT}^{\text{exp}}(\Delta t)$  (left bottom) as a function of  $\Delta t$  (solid line); dashed lines correspond to  $\pm 10\%$  uncertainty on  $\eta_{3\pi^0}$ ; the constant  $D$  has been fixed to one for simplicity. Zoomed plots for  $\Delta t > 0$  (right).

### 4.3 CPT symmetry test

For the direct CPT symmetry test one can define the following ratios of probabilities, similarly as for the  $T$  test:

$$\begin{aligned}
R_{1,CPT}(\Delta t) &= P[K^0(0) \rightarrow K_+(\Delta t)] / P[K_+(0) \rightarrow \bar{K}^0(\Delta t)] \\
R_{2,CPT}(\Delta t) &= P[K^0(0) \rightarrow K_-(\Delta t)] / P[K_-(0) \rightarrow \bar{K}^0(\Delta t)] \\
R_{3,CPT}(\Delta t) &= P[\bar{K}^0(0) \rightarrow K_+(\Delta t)] / P[K_+(0) \rightarrow K^0(\Delta t)] \\
R_{4,CPT}(\Delta t) &= P[\bar{K}^0(0) \rightarrow K_-(\Delta t)] / P[K_-(0) \rightarrow K^0(\Delta t)] .
\end{aligned} \tag{6}$$

The measurement of any deviation from the prediction  $R_{i,CPT}(\Delta t) = 1$  imposed by CPT invariance is a signal of CPT violation. At a  $\phi$ -factory the corresponding observable quantities are, for  $\Delta t > 0$ :

$$\begin{aligned}
R_{2,CPT}^{\text{exp}}(\Delta t) &\equiv \frac{I(\ell^-, 3\pi^0; \Delta t)}{I(\pi\pi, \ell^-; \Delta t)} = R_{2,CPT}(\Delta t) \times D_{CPT} \\
R_{4,CPT}^{\text{exp}}(\Delta t) &\equiv \frac{I(\ell^+, 3\pi^0; \Delta t)}{I(\pi\pi, \ell^+; \Delta t)} = R_{4,CPT}(\Delta t) \times D_{CPT}
\end{aligned} \tag{7}$$

while for  $\Delta t < 0$ :

$$R_{2,CPT}^{\text{exp}}(\Delta t) = \frac{D_{CPT}}{R_{1,CPT}(|\Delta t|)} , \quad R_{4,CPT}^{\text{exp}}(\Delta t) = \frac{D_{CPT}}{R_{3,CPT}(|\Delta t|)} . \tag{8}$$

Here the normalization constant  $D_{CPT}$  is  $D_{CPT} = \{\text{BR}(K_L \rightarrow 3\pi^0) \cdot \Gamma_L\} / \{\text{BR}(K_S \rightarrow \pi\pi) \cdot \Gamma_S\}$  without any assumption on CPT violation in semileptonic decays.

The KLOE-2 experiment could make a statistically significant CPT test, measuring the ratios  $R_{2,CPT}^{\text{exp}}(\Delta t)$  and  $R_{4,CPT}^{\text{exp}}(\Delta t)$  integrated in the statistically most populated  $\Delta t$  region,  $0 \leq \Delta t \leq 300 \tau_S$  [5]. In this region  $R_{2,CPT}^{\text{exp}}(\Delta t)$  and  $R_{4,CPT}^{\text{exp}}(\Delta t)$  are expected to be constant. Here CPT violation is proportional to  $\text{Re } \delta$ , which do not vanish in the limit  $\Delta\Gamma \rightarrow 0$ , escaping the previous controversy for the T test. Most importantly the impact of direct  $CP$  violation on the orthogonality condition is completely negligible in this  $\Delta t$  region (see Fig.2 right), and does not affect the significance of the CPT test.

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